

Structure of Split Supersymmetry and Simple Models

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Abstract

We derive in detail a condition on the Higgs mass parameters that is necessary for the recently proposed “split supersymmetry” (split SUSY) scenario to provide a realistic magnitude of $\tan \beta$. The nature of this condition can be understood by showing how the Higgs sector of the minimal supersymmetric Standard Model reduces to that of the Standard Model in the heavy limit of the soft supersymmetry breaking Higgs mass parameters. Based on this condition, we present some simple supersymmetry breaking models that each provides a realistic split-SUSY mass spectrum, in accordance with the scale of the gravitino mass ($m_{3/2}$) in relation to those of the soft scalar mass (\tilde{m}) and the gaugino mass ($M_{1/2}$) employed in each, namely $m_{3/2} \geq \tilde{m}$, $\tilde{m} \geq m_{3/2} \geq M_{1/2}$ and $M_{1/2} \geq m_{3/2}$, respectively, with the relation $\tilde{m} \gg M_{1/2}$ of the split-SUSY mass spectrum.

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1 Introduction

The Standard Model (SM), with a simple extension to incorporate neutrino masses and mixings, is in good agreement with almost of all current experimental data. However, the SM contains the gauge hierarchy problem of quantum field theory. This results from the quadratic divergence of the Higgs boson mass on the new physics scale arising in quantum theories, which makes a very precise fine-tuning necessary in order to realize the correct electroweak scale if this new physics scale lies at a high energy scale, such as the scale of the grand unified theory (GUT) or the Planck scale. In other words, the vacuum in the SM is not stable with respect to quantum corrections. It is well known that this fine-tuning problem can be solved by introducing supersymmetry (SUSY) [1]. The minimally extended SUSY SM (MSSM) has the elegant feature of gauge coupling unification, and for this reason, many people believe that there exists a 4-dimensional SUSY. Some people also believe that SUSY is required for the construction of a quantum theory of gravity.

However, SUSY particle has not yet been observed experimentally. Also, the proton-decay predicted by SUSY GUT models has not yet been observed [2]. Given this situation, we might consider the possibility of heavy SUSY particles as one possibility. Recently, the split supersymmetry (split SUSY) scenario was proposed [3, 4]. In this scenario, nature is fine-tuned intrinsically, and SUSY has nothing to do with the gauge hierarchy problem. The scalar masses are super heavy, while the fermion masses are maintained at the electroweak scale, protected by the chiral ($U(1)_R$) symmetry. In this way, the split SUSY scenario forgets the fine-tuning problem originating in the Higgs mass quadratic divergence. Related studies are given in Ref. [5].

In this paper, we first overview the fact that the Higgs sector of the MSSM reduces to that of the SM in the heavy limit of the soft SUSY breaking Higgs mass parameters. This means that split SUSY is just the MSSM containing super-heavy scalar masses. We examine the Higgs potential in detail and derive a condition on the Higgs mass parameters ($|m_u^2| \sim |m_d^2| \sim |B\mu|$) that is necessary for the split-SUSY scenario to yield suitable values of $\tan \beta$, i.e., those in the range $1 \leq \tan \beta \leq 60$. Then, we present some simple supersymmetry breaking models that provide realistic split-SUSY mass spectra, in accordance with the relations among the scales of the gravitino mass ($m_{3/2}$), the soft scalar mass (\tilde{m}) and the gaugino mass ($M_{1/2}$) used in each, namely $m_{3/2} \geq \tilde{m}$, $\tilde{m} \geq m_{3/2} \geq M_{1/2}$ and $M_{1/2} \geq m_{3/2}$, respectively, along with the condition $\tilde{m} \gg M_{1/2}$ of the split-SUSY mass spectrum.

2 Structure of split-SUSY

Let us present the detailed structure of the Higgs potential in split SUSY. This is the same as the minimal SUSY standard model (MSSM) with super-heavy soft masses. Here we explicitly give the fine-tuning conditions required in the Higgs potential of split SUSY.

The standard model (SM) Higgs potential is given by

$$V_{\text{SM}} = -m^2 h^\dagger h + \frac{\lambda}{2} (h^\dagger h)^2, \quad (1)$$

while that of the MSSM is given by

$$V_{\text{MSSM}} = m_u^2 |H_u|^2 + m_d^2 |H_d|^2 + (B\mu \epsilon_{ij} H_u^i H_d^j + \text{h.c.})$$

$$+\frac{g^2+g'^2}{8}(|H_u|^2-|H_d|^2)+\frac{g^2}{2}|H_u|^2|H_d|^2-\frac{g^2}{2}|\epsilon_{ij}H_u^iH_d^j|^2, \quad (2)$$

where $m_u^2 = |\mu|^2 + \tilde{m}_u^2$ and $m_d^2 = |\mu|^2 + \tilde{m}_d^2$. The masses $\tilde{m}_{u,d} \simeq \tilde{m}$ are the soft SUSY breaking masses of the up-type and down-type Higgs doubles. We make $B\mu$ real and positive through field redefinitions. Each neutral component of the Higgs doublets develops a vacuum expectation value (VEVs), $\langle H_u \rangle = v \sin \beta / \sqrt{2}$ and $\langle H_d \rangle = v \cos \beta / \sqrt{2}$. In the split-SUSY scenario, the threshold corrections to the quartic coupling are small, due to the smallness of the A terms, and the energy scale dependence of the quartic coupling should be estimated using the renormalization group equation analysis rather than the effective potential [3]. Thus, obtaining the tree-level Higgs potential is enough for the analysis at the high energy scale of \tilde{m} . As shown below, because of the large soft SUSY breaking terms, only the SM-like Higgs scalar survives at low energies, and the MSSM Higgs potential is reduced to the SM one.

Necessary and sufficient conditions for realizing a suitable electroweak symmetry breaking are given by [3]

$$\begin{aligned} m_u^2 + m_d^2 - 2B\mu &> 0, & (m_u^2 + m_d^2)^2 &< (m_u^2 - m_d^2)^2 + (2\mu B)^2, \\ (m_u^2 + m_d^2 + m_{EW}^2)^2 &> (m_u^2 - m_d^2)^2 + (2\mu B)^2, \end{aligned} \quad (3)$$

where m_{EW} is the electroweak mass scale, which is $\mathcal{O}(10^2)$ GeV. The third condition implies that the magnitude of the negative mass squared eigenvalue should be the electroweak scale. It is well known that the minimization conditions $dV/dH_{u,d} = 0$ can be expressed as

$$\sin 2\beta = \frac{2B\mu}{m_u^2 + m_d^2}, \quad M_Z^2 = \frac{m_u^2 - m_d^2}{\cos 2\beta} - (m_u^2 + m_d^2). \quad (4)$$

We now introduce a field redefinition, employing $H_1 \equiv (\epsilon H_d^*)$ and $H_2 \equiv H_u$. Then, in the basis (H_1, H_2) , the Higgs mass matrix is obtained as

$$\begin{pmatrix} m_d^2 & B\mu \\ B\mu & m_u^2 \end{pmatrix} = \begin{pmatrix} m_A^2 \sin^2 \beta - \frac{1}{2}M_Z^2 \cos(2\beta) & m_A^2 \sin \beta \cos \beta \\ m_A^2 \sin \beta \cos \beta & m_A^2 \cos^2 \beta + \frac{1}{2}M_Z^2 \cos(2\beta) \end{pmatrix}, \quad (5)$$

by using Eq. (4). Here we have $m_A^2 \equiv m_u^2 + m_d^2$. This mass matrix automatically satisfies the conditions in Eq. (3) with the identification $m_{EW} = M_Z$.

In the split-SUSY scenario, one linear combination of the Higgs doublet,

$$\tilde{h} = -\cos \beta H_1 + \sin \beta H_2, \quad (6)$$

is light and only survives below the energy scale of $\mathcal{O}(\tilde{m})$, as shown in Appendix A. (The eigenstate orthogonal to \tilde{h} is $\tilde{H} = -\sin \beta H_1 - \cos \beta H_2$.) For this reason, the low energy effective theory should be written in terms of \tilde{h} only. The effective Higgs potential is obtained from Eqs. (2) and (6) as

$$V_{\text{MSSM}}^{\text{eff}} = -m'^2 |\tilde{h}|^2 + \frac{\lambda'}{2} |\tilde{h}|^4, \quad \left(\lambda' = \frac{g^2 + g'^2}{4} \cos^2 2\beta \right), \quad (7)$$

³In the basis (\tilde{h}, \tilde{H}) , Eq. (5) is rewritten as $\begin{pmatrix} -\frac{1}{2}M_Z^2 \cos^2(2\beta) & -\frac{M_Z^2}{4} \sin(4\beta) \\ -\frac{M_Z^2}{4} \sin(4\beta) & m_A^2 + \frac{M_Z^2}{4}(1 + \cos(4\beta)) \end{pmatrix}$.

where $m'^2 \equiv M_Z^2 \cos^2(2\beta)$ [see Eq. (32)]. The Higgs mass becomes zero (resp., M_Z) when $\tan\beta = 1$ (resp., $\beta = \pi/2$) at the high energy scale of $\mathcal{O}(\tilde{m})$. This can also be understood by considering the effective quartic coupling, λ' , in Eq. (7) as follows. When $\tan\beta = 1$, λ' is zero, and therefore the Higgs mass $\sqrt{\lambda}v$ vanishes at the SUSY breaking scale, and only the radiative corrections induce a finite Higgs mass at the low energy. Contrastingly, when $\beta = \pi/2$, the Higgs mass becomes M_Z at the SUSY breaking scale, and in this case also, the mass is increased by the radiative corrections. This is the reason why $\cos 2\beta = 0$ (resp., $\cos 2\beta = -1$) is found to have the smallest (resp., largest) mass of the low energy physical Higgs scalar, h^0 , in Ref. [3].

We have shown that the MSSM Higgs potential reduces to the SM one when the soft scalar masses, \tilde{m} , are much larger than the electroweak scale. Explicitly, the Higgs doublet, \tilde{h} , is the direction of the VEV and also contains all would-be NG bosons and one SM-like physical Higgs scalar. The important point is that the vacuum stability conditions in Eq. (4) must be satisfied even when we introduce super-heavy soft masses. It should be noted that the conditions given in Eq. (4) are essential and that these represent the fine-tuning required in the split-SUSY scenario. Some examples of the split-SUSY scenario given in Ref. [3] suggest scalar masses of $\mathcal{O}(10^{12-13})$ GeV (which is the scale favored by the cosmological considerations), while $B\mu$ is suppressed by the chiral $[U(1)_R]$ symmetry. However, in this case, there is an extremely large $\tan\beta \sim m_d^2/B\mu$, as seen from Eqs. (4) and (5), and therefore it is difficult to obtain a realistic bottom quark Yukawa coupling.

⁴ In order to obtain a realistic value of $\tan\beta$, $B\mu$ should be of the same order as the Higgs mass. It is non-trivial to construct a model that can naturally provide a realistic split-SUSY mass spectrum.

3 Simple models

In this section we present simple SUSY breaking models satisfying the condition $|m_u^2| \sim |m_d^2| \sim |B\mu|$, which is necessary for a realistic split-SUSY scenario with $1 \leq \tan\beta \leq 60$, as shown in the previous section. There are many possible ways to construct such models. We consider several models characterized by the scale of the gravitino mass, $m_{3/2}$, in comparison with the soft scalar mass, \tilde{m} and the gaugino mass $M_{1/2}$, with the split-SUSY mass spectrum, $\tilde{m} \gg M_{1/2}$.

1. Case of large gravitino mass ($0.01 m_{3/2} > M_{1/2}$)

In this case, we should first note the gaugino mass generated through the superconformal anomaly (anomaly mediation) [7, 8], which is approximated as $(\alpha_{SM}/4\pi)F_\phi \simeq 0.01F_\phi$, where α_{SM} is the gauge coupling in the Standard Model, and F_ϕ is the F-term of the compensating multiplet. In normal SUSY breaking scenarios in SUGRA, we obtain $F_\phi \simeq m_{3/2}$, where $m_{3/2}$ is the gravitino mass. Therefore, for a SUSY breaking model with gravitino mass satisfying $0.01m_{3/2} > M_{1/2}$, a mechanism that can suppress the anomaly mediation is necessary to realize the split-SUSY scenario. Such a model is the ‘‘almost no-scale’’ SUGRA

⁴There is a finite quantum correction for the bottom quark mass, which is produced through the anti-holomorphic Yukawa interaction induced by the gluino and higgsino 1-loop diagrams [6]. However, this correction is negligibly small, due to the super-heavy masses of the sfermions in the split-SUSY scenario.

model [9, 10], whose structure is in fact crucial in the split-SUSY scenario, as shown in the original paper [3].

Let us first introduce the almost no-scale model. Although Refs. [9, 10] consider extra-dimensional theories, a 4D SUGRA model can yield the same type of a structure as these extra-dimensional theories if we allow fine-tuning of the parameters in the Kahler potential. We consider the SUGRA Lagrangian,

$$\mathcal{L} = \int d^4\theta \mathcal{K}(z^\dagger, z) \phi^\dagger \phi + \left\{ \int d^2\theta \phi^3 W_0 + \text{h.c.} \right\}, \quad (8)$$

with the Kahler potential

$$\mathcal{K}(z^\dagger, z) = -3M_4^2(z + z^\dagger + \epsilon f(z, z^\dagger)), \quad (9)$$

where z and ϕ denote a hidden sector (dilaton) superfield and a compensating multiplet ($\phi = 1 + F_\phi$), respectively, ϵ is a small dimensionless parameter, M_4 is the 4D Planck scale, and W_0 is a constant superpotential. The original (4D) no-scale model [11] is obtained in the limit $\epsilon \rightarrow 0$. The equations of motion for the auxiliary fields, $d\mathcal{L}/dF_\phi^\dagger = 0$ and $d\mathcal{L}/dF_z^\dagger = 0$, lead to

$$F_\phi \simeq -\frac{W_0^\dagger}{M_4^2} \epsilon f_{z^\dagger z} = -\epsilon m_{3/2} f_{z^\dagger z}, \quad F_z \simeq \frac{W_0^\dagger}{M_4^2} = m_{3/2}, \quad (10)$$

for small ϵ . Here, $f_{z^\dagger z}$ stands for $\partial^2 f(z^\dagger, z)/\partial z^\dagger \partial z$. Then, the scalar potential is given by

$$V = -3F_\phi W_0 \simeq 3 \frac{|W_0|^2}{M_4^2} \epsilon f_{z^\dagger z} = 3\epsilon m_{3/2}^2 M_4^2 f_{z^\dagger z} \quad (11)$$

Assuming that $f_{z^\dagger z}$ has the form $f_{z^\dagger z} = (|z|^2 - 1/4)^2 - 1$, for example, the potential has a minimum at $\langle z \rangle = 1/2$, with potential energy

$$V_{\min} \simeq -3\epsilon m_{3/2}^2 M_4^2. \quad (12)$$

Here, the almost no-scale structure is realized; that is, we $F_\phi \simeq \epsilon m_{3/2} \ll m_{3/2}$.

Of course, the contact terms among the gauginos in the visible sector and z should be suppressed in order to realize the split-SUSY scenario. However, in this model, it is difficult to find a symmetry that would forbid such contact terms. A simple way to avoid this problem is to introduce the sequestering scenario [7], in which we assume that the dilaton sector and the visible sector exist on different branes that are spatially separated in the extra-dimensions.⁵ Because the contribution from the anomaly mediation is sub-dominant, an additional SUSY breaking source and a SUSY breaking mediation mechanism must be introduced in order to realize a split-SUSY mass spectrum. For this purpose, consider a hidden sector with a $U(1)$ gauge symmetry and the Fayet-Iliopoulos D-term with particles X and Y , which have $U(1)$ charges 1 and -1 , respectively. Suppose that these hidden sector fields exist on the visible sector brane and that there exists a superpotential $W = mXY$.

⁵In such a scenario, one of the most important points is radius stabilization, since it is, in general, very closely related to SUSY breaking and its mediation mechanism. With regard to this point, the models proposed in [12] are noteworthy, because in them, radius stabilization is realized independently of the SUSY breaking and its mediation mechanism.

Together with the dilaton sector, the total Lagrangian (in the 4D effective theory) is given by

$$\begin{aligned}\mathcal{L} = & \int d^4\theta \left[\mathcal{K}(z^\dagger, z) \phi^\dagger \phi + X^\dagger e^{+2gV} X + Y^\dagger e^{-2gV} Y \right] \\ & + \left[\int d^2\theta \left(\phi^3 W_0 + \phi mXY \right) + \text{h.c.} \right] \\ & + \left[\frac{1}{4} \int d^2\theta \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.} \right] + \int d^4\theta \xi^2 V,\end{aligned}\tag{13}$$

where the last term is the Fayet-Iliopoulos D-term, and ξ is a real parameter with dimension of mass ($\xi^2 > 0$ with our definition of the $U(1)$ charge). If we consider an anomalous $U(1)$ gauge theory, the parameter can be understood as $\xi^2 = g_S^2 M_S^2 \text{Tr} Q / 192\pi^2$ with the string coupling g_S , the string scale M_S , and the anomalous $U(1)$ charge Q [13]. Here, the superfields X and Y have been rescaled as $X, Y \rightarrow X/\phi, Y/\phi$, so that the compensating multiplet ϕ disappears in the Kahler potential for X and Y .

Note that the dilaton sector (the almost no-scale sector) and the $U(1)$ gauge sector are decoupled in the Kahler potential. Because of this fact, in the equations of motion for the auxiliary fields in the almost no-scale sector, W_0 is simply replaced by $W_0 + 1/3mXY$. Thus, if $|W_0| \gg \langle |mXY| \rangle$, the structure of the almost no-scale sector remains almost the same. It must be noted, though, that non-zero F_ϕ induced in the almost no-scale sector affects the scalar potential for X and Y . However, if $F_\phi \simeq \epsilon m_{3/2} \ll m$, the scalar potential of the $U(1)$ gauge sector in SUGRA is almost the same as that in the global SUSY limit ($F_\phi \rightarrow 0$), because the scalar potential for X and Y is controlled by the scale m . This is the case that we examine in the following.

Analyzing the potential in the $U(1)$ gauge sector (ignoring F_ϕ), we find

$$\langle X \rangle = 0, \quad \langle Y \rangle = \pm \sqrt{\xi^2 - \frac{m^2}{g^2}}\tag{14}$$

and

$$\langle F_X \rangle = m \langle Y \rangle, \quad \langle F_Y \rangle = 0, \quad \langle D \rangle = \frac{m^2}{g},\tag{15}$$

for $\xi^2 > m^2/g^2$. In the following, we assume $\xi^2 \gg m^2/g^2$ for simplicity. Then the potential energy is found to be

$$V_{\min} = m^2 \langle Y \rangle^2 + \frac{1}{2} \langle D \rangle^2 \simeq m^2 \xi^2 > 0.\tag{16}$$

In order to obtain a vanishing cosmological constant, this potential energy should be canceled out by the negative contribution in the almost no-scale sector, given by Eq. (12), and hence we have

$$m^2 \xi^2 \simeq \epsilon m_{3/2}^2 M_4^2.\tag{17}$$

We next consider the soft SUSY breaking mass spectrum. We impose R-parity with the usual assignments for the MSSM fields and even for the other fields. The values of the scalar soft mass squared for the MSSM particles, represented by Ψ , are determined by

$$\left[\frac{X^\dagger X}{M_4^2} \Psi^\dagger \Psi \right]_D \simeq m^2 \left(\frac{\xi}{M_4} \right)^2 |\tilde{\Psi}|^2.\tag{18}$$

while, for the gaugino mass, we have

$$\left[\frac{XY}{M_4^2} \text{tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) \right]_F \simeq m \left(\frac{\xi}{M_4} \right)^2 \lambda \lambda. \quad (19)$$

Furthermore, the μ -term can be obtained from

$$\left[\frac{X^\dagger Y^\dagger}{M_4^2} H_u H_d \right]_{F^\dagger} \simeq m \left(\frac{\xi}{M_4} \right)^2 H_u H_d, \quad (20)$$

while the $B\mu$ term is obtained as

$$\left[\frac{X^\dagger X}{M_4^2} H_u H_d \right]_D \simeq m^2 \left(\frac{\xi}{M_4} \right)^2 H_u H_d. \quad (21)$$

Note that the relations $\tilde{m}^2 \simeq B\mu$ and $M_{1/2} \simeq \mu$ are automatically realized, because of the $U(1)$ gauge invariance and the holomorphy of the gauge kinetic function and the superpotential. If we tune the parameters such that $\xi/M_4 = \delta \ll 1$, the split-SUSY mass spectrum, $\tilde{m} \simeq m\delta \gg M_{1/2} \sim m\delta^2$, is realized. In this case, the condition given in Eq. (17) implies the relations $m_{3/2} \simeq \tilde{m}/\sqrt{\epsilon} \gg \tilde{m}$. In summary, the above model leads to a realistic split-SUSY mass spectrum satisfying $m_{3/2} \gg \tilde{m} \simeq \sqrt{B\mu} \gg M_{1/2} \simeq \mu$ under the condition $0.01\epsilon m_{3/2} \leq M_{1/2} = 100\text{GeV} - 1\text{TeV}$, for negligible anomaly mediation contributions. In the original paper, Ref. [3], the same split-SUSY mass spectrum is obtained the basis of extra-dimensional models. In the following, we show that the model studied here has more flexibility and can lead to various split-SUSY mass spectra.

In general, we can introduce the usual (tree level) μ -term into the model as in the MSSM, that is, in the form

$$\int d^2\theta \phi^3 \mu_{\text{tree}} H_u H_d. \quad (22)$$

Although this μ parameter can take any values,⁶ note that, once the μ -term exists, the relations $B\mu \simeq F_\phi \mu_{\text{tree}} \simeq \epsilon m_{3/2} \mu_{\text{tree}}$ ⁷ is induced in SUGRA. Thus the total μ parameter (μ_{total}) and the total $B\mu$ ($(B\mu)_{\text{total}}$) are given by $\mu_{\text{total}} \simeq M_{1/2} + \mu_{\text{tree}}$ and $(B\mu)_{\text{total}} \simeq \tilde{m}^2 + \epsilon m_{3/2} \mu_{\text{tree}} \simeq \tilde{m}^2 + \sqrt{\epsilon} \tilde{m} \mu_{\text{tree}}$, respectively. In the case $\mu_{\text{tree}} \leq M_{1/2}$, we obtain the above result. For the opposite case, $\mu_{\text{tree}} \geq M_{1/2}$, the condition for a realistic split-SUSY scenario, namely $\tilde{m}^2 + \mu_{\text{total}}^2 \sim (B\mu)_{\text{total}}$, leads to $\mu_{\text{tree}} \leq \tilde{m}$ and we obtain the mass spectrum $m_{3/2} \gg \tilde{m} \gg M_{1/2}$ with μ_{tree} satisfying $\tilde{m} \geq \mu_{\text{tree}} \geq M_{1/2}$.

It is possible to extend our model to the case in which the MSSM particles have non-zero $U(1)$ charges. We now show that this extended model can lead to a split-SUSY mass spectrum that differs from that given above. Assume that $\xi \sim m$ with $g \sim 1$, for simplicity. In this case, the scalars in the MSSM acquire mass through the VEV of D-term, $\tilde{m}^2 = gq\langle D \rangle \sim qm^2$, with the $U(1)$ charges. The condition given in Eq. (17) implies

$$\tilde{m} \simeq m \simeq \frac{\sqrt{\epsilon}}{\delta} m_{3/2}. \quad (23)$$

⁶In the split-SUSY scenario, it may not be so clear whether the well-known μ -problem is really a problem, since this scenario is insensitive to fine-tunings.

⁷Here we have denoted the μ parameter at the tree level as μ_{tree} to avoid confusion with the μ -term obtained in Eq. (20).

The gaugino mass is, again, obtained from Eq. (19), which yields $M_{1/2} \sim m\delta^2$, and thus we find $M_{1/2} \sim \sqrt{\epsilon}\delta m_{3/2}$ from the above equation. For $\sqrt{\epsilon}/\delta \geq 1$ ($\sqrt{\epsilon}/\delta \leq 1$), we obtain the mass spectrum $\tilde{m} \geq m_{3/2} \gg M_{1/2}$ ($m_{3/2} \geq \tilde{m} \gg M_{1/2}$).

However, there is a problem in the above: $B\mu$ obtained from Eq. (21) is much smaller than \tilde{m}^2 , and therefore the condition for the realistic split-SUSY scenario cannot be satisfied. Unfortunately, the new contribution $B\mu \simeq \epsilon m_{3/2} \mu_{\text{tree}}$ from the tree level μ term cannot resolve this problem. This follows from the condition for a realistic split-SUSY scenario, $\tilde{m}^2 + \mu_{\text{tree}}^2 \sim \epsilon m_{3/2} \mu_{\text{tree}}$. We cannot find any μ_{tree} satisfying this condition. A simple way to ameliorate the problem is to introduce an additional contribution to $B\mu$. Let us consider a Polonyi model with the superpotential $W = M^2\Phi$. We choose a special Kahler potential for Φ so that the Polonyi model leads to $\langle\Phi\rangle \simeq 0$ and $F_\Phi \simeq M^2$. In order not to change the structure of the $U(1)$ gauge sector, M^2 must satisfy the condition $M^2 \simeq F_\Phi \leq F_X \simeq m^2$. Then, we introduce the superpotential for the Higgs sector $W_H = \Phi H_u H_d$, which leads to $B\mu \simeq M^2$. Then, tuning the parameter to realize $M^2 \simeq \tilde{m}^2 + \mu_{\text{tree}}^2$, we obtain realistic split-SUSY mass spectra in both cases, $\tilde{m} \geq m_{3/2}$ and $m_{3/2} \geq \tilde{m}$, with various values of μ_{tree} satisfying this condition.

As discussed above, in the case with a tree level μ -term, we can take the μ parameter to be much larger than the gaugino mass. This implies a mass spectrum different from the originally proposed in Ref. [3]. The phenomenology of the split-SUSY scenario with such a large μ -parameter is investigated in Ref. [14]. However, we note that, once a large μ term is introduced, the Higgs superfields play the role of the ‘‘messengers’’ in the gauge mediated SUSY breaking model (gauge mediation) [15], and as a result, the gauginos (wino and bino) acquire soft masses of the order of $(\alpha_{SM}/4\pi)B\mu/\mu$. Hence, the scale of μ is limited in order to keep the gaugino masses near the electroweak scale.

2. Case of small gravitino mass ($0.01 m_{3/2} < M_{1/2}$)

In this case, the contribution from the anomaly mediation is small, and therefore the almost no-scale structure is no longer necessary. The split-SUSY mass spectrum in this case implies $\tilde{m} \gg m_{3/2}$. Therefore, a SUSY breaking mediation mechanism other than the SUGRA mediation should have the dominant contribution to sparticle masses. Again, let us consider the $U(1)$ gauge model, in which the MSSM matter and Higgs superfields have non-zero $U(1)$ charges. As discussed above, large values of the soft mass squared for the scalars in the MSSM are induced through the $U(1)$ D-term. The main difference between this and the previous model is that here, the almost no-scale structure is no longer necessary. The soft mass spectrum in this case is obtained by taking $\epsilon \sim 1$ in the previous results, Eqs. (17) and (19); this yields $\tilde{m} \simeq m \gg m_{3/2} \simeq m\delta \gg M_{1/2} \sim m\delta^2$. New contributions to the gaugino masses are necessary to realize the case $M_{1/2} > m_{3/2}$. In order to obtain these contributions, let us introduce the gauge mediation sector into the model [15]. Consider a simple messenger sector given by

$$W_m = (M_m + F_m \theta^2) (\lambda_Q \overline{Q} Q + \lambda_\ell \overline{\ell} \ell), \quad (24)$$

where \overline{Q} , Q , $\overline{\ell}$ and ℓ are the vector-like messenger quarks and leptons, respectively. The MSSM gaugino masses are generated through one-loop diagram of the messenger fields as

$$M_{1/2} \simeq \left(\frac{\alpha_{SM}}{4\pi} \right) \frac{F_m}{M_m}. \quad (25)$$

If $\langle F_X \rangle \geq F_m$, the gravitino mass retains the same value, while the gaugino mass can become larger than the gravitino mass through the gauge mediation when the messenger scale, M_m , is small enough. Choosing appropriate values for M_m and F_m in the messenger sector, we can realize the split-SUSY mass spectrum $\tilde{m} \gg M_{1/2} \gg m_{3/2}$ with a small gravitino mass.

For the Higgs mass parameters, we can use the chiral superfield $M_m + F_m \theta^2$ in the messenger sector and introduce the Higgs superpotential

$$W_H = (M_m + F_m \theta^2) H_u H_d, \quad (26)$$

in addition to the μ -term at the tree level. Thus, we obtain $\mu_{\text{total}} = M_m + \mu_{\text{tree}}$ and $B\mu \simeq F_m$. A realistic split-SUSY mass spectrum requires $\tilde{m}^2 + (M_m + \mu_{\text{tree}})^2 \simeq B\mu \simeq F_m$, which can be rewritten as $\tilde{m}^2 + (M_m + \mu_{\text{tree}})^2 \simeq 100 M_{1/2} M_m$ by using Eq. (25). In order to realize the hierarchy $\tilde{m} \gg 100 M_{1/2}$ ($M_m \gg 100 M_{1/2}$), a cancellation between M_m and μ_{tree} is necessary so as to satisfy $(M_m + \mu_{\text{tree}})^2 \leq 100 M_{1/2} M_m$. Once this fine-tuning is realized, we can obtain a realistic split-SUSY mass spectrum with $\tilde{m} \simeq B\mu \gg M_{1/2}$, with the μ parameter satisfying $\tilde{m} \geq \mu \geq M_{1/2}$.

4 Summary

In a recently proposed split-SUSY scenario [3], the scalar masses are very large, while the fermion masses, protected by chiral symmetry, are set to the electroweak scale. In this paper, we have shown, in detail, how the Higgs sector of the MSSM is reduced to that of the SM in the limit of large soft SUSY breaking Higgs mass parameters. Then, we demonstrated that the conditions $|m_u^2| \sim |m_d^2| \sim |B\mu|$ are necessary to obtain a suitable magnitude of $\tan \beta$ that yields a realistic bottom quark Yukawa coupling. Based on these conditions, we have presented some simple models that provide realistic split-SUSY mass spectra for various gravitino mass scales, from $m_{3/2} \gg \tilde{m}$ to $M_{1/2} \gg m_{3/2}$.

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A Mass spectra in split-SUSY (MSSM)

In this appendix, we show that \tilde{h} can be regarded as the SM Higgs doublet in the split-SUSY scenario by determining the masses of the charged and neutral sectors. The Higgs doublets are written

$$H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}(v \sin \beta + \eta_u + i\zeta_u) \end{pmatrix}, \quad H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v \cos \beta + \eta_d + i\zeta_d) \\ H_d^- \end{pmatrix}, \quad (27)$$

and we carry out a field redefinition through which we introduce $H_1^T \equiv (\epsilon H_d^*)^T = (H_d^+, -\frac{1}{\sqrt{2}}(v \cos \beta + \eta_d - i\zeta_d))$ and $H_2 \equiv H_u$, as in section 2. Equation (27) suggests that the direction of the VEV is the same as that of \tilde{h} , defined in Eq. (6). Therefore, the light field \tilde{h} is just a linear combination of the fields acquiring the VEV. The direction of the would-be NG bosons (W^\pm, Z) is also the same as that of \tilde{h} . This can be understood by considering the following charged and pseudo-scalar mass matrices:

$$-\mathcal{L}_{\text{charged}} = (\tilde{m}^2 + M_W^2)(H_d^-, H_u^-) \begin{pmatrix} \sin^2 \beta & \sin \beta \cos \beta \\ \sin \beta \cos \beta & \cos^2 \beta \end{pmatrix} \begin{pmatrix} H_d^+ \\ H_u^+ \end{pmatrix}, \quad (28)$$

$$-\mathcal{L}_{\text{pseudo}} = \frac{\tilde{m}^2}{2}(\zeta_d, \zeta_u) \begin{pmatrix} \sin^2 \beta & \sin \beta \cos \beta \\ \sin \beta \cos \beta & \cos^2 \beta \end{pmatrix} \begin{pmatrix} \zeta_d \\ \zeta_u \end{pmatrix}. \quad (29)$$

These matrices suggest that the physical charged (pseudo-scalar) Higgs is in the direction of \tilde{H} , and its mass is $m_A^2 + M_W^2$ (m_A^2). This mass is $\mathcal{O}(\tilde{m})$, which is too large to survive at low energy in the split-SUSY scenario. On the other hand, the mass matrix of the neutral scalar is given by

$$\begin{aligned} -\mathcal{L}_{\text{scalar}} &= \frac{1}{2}(-\eta_d, \eta_u) \begin{pmatrix} m_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta & (m_A^2 + M_Z^2) \sin \beta \cos \beta \\ (m_A^2 + M_Z^2) \sin \beta \cos \beta & m_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta \end{pmatrix} \begin{pmatrix} -\eta_d \\ \eta_u \end{pmatrix} \\ &= \frac{m_A^2}{2}(-\eta_d, \eta_u) \begin{pmatrix} \sin^2 \beta & \sin \beta \cos \beta \\ \sin \beta \cos \beta & \cos^2 \beta \end{pmatrix} \begin{pmatrix} -\eta_d \\ \eta_u \end{pmatrix} + \mathcal{O}(M_Z^2). \end{aligned} \quad (30)$$

This implies that the mass matrix can be diagonalized by the same linear combination *to a good approximation* in the split-SUSY scenario. In the case $M_Z^2 \ll m_A^2$, the light and heavy neutral Higgs eigenstates, h^0 and H^0 , are given by $h^0 = -\cos \beta(-\eta_d) + \sin \beta \eta_u$ and $H^0 = \sin \beta(-\eta_d) + \cos \beta \eta_u$, respectively, which means that the light neutral scalar h^0 is included in \tilde{h} . Without the approximation, Eq. (30) becomes

$$\begin{pmatrix} M_Z^2 \cos^2(2\beta) & \frac{M_Z^2}{2} \sin(4\beta) \\ \frac{M_Z^2}{2} \sin(4\beta) & m_A^2 + \frac{M_Z^2}{2}(1 - \cos(4\beta)) \end{pmatrix} \quad (32)$$

in the basis (h^0, H^0) . Because the off-diagonal elements are $\mathcal{O}(M_Z^2)$, the heavy field H^0 with mass m_A decouples, and only h^0 survives at low energy, having mass $M_Z^2 \cos^2(2\beta)$ when $M_Z^2 \ll m_A^2$.

In summary, we have shown that \tilde{h} is in the direction as the VEV and containing would-be NG boson and also a light scalar. Therefore, we can conclude that \tilde{h} corresponds to the SM Higgs doublet, which is what we sought to show in this appendix. In the split-SUSY scenario, \tilde{H} is decoupled, and only \tilde{h} survives at low energy.

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